

# پاسخنامه تشریحی

$$\left. \begin{aligned} x_M &= \frac{x_A + x_B}{2} = \frac{14 + 10}{2} \rightarrow x_M = 12 \\ y_M &= \frac{y_A + y_B}{2} = \frac{3 - 13}{2} \rightarrow y_M = -5 \end{aligned} \right\} \rightarrow M(12, -5)$$

$$OM = \sqrt{x_M^2 + y_M^2} = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} \rightarrow OM = 13$$

$$x_M = \frac{x_A + x_B}{2} = \frac{3 + 5}{2} \Rightarrow x_M = 4 \rightarrow m(4, 3)$$

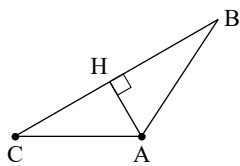
$$y_M = \frac{y_A + y_B}{2} = \frac{7 - 1}{2} \Rightarrow y_M = 3$$

$$m_{AB} = \frac{y_A - y_B}{x_A - x_B} = \frac{7 - (-1)}{3 - 5} = \frac{8}{-2} \rightarrow m_{AB} = -4$$

$$m \cdot m_{AB} = -1 \rightarrow m(-4) = -1 \rightarrow m = \frac{1}{4} \quad \text{شیب خط عمودمنصف}$$

$$\text{معادله‌ی عمودمنصف: } y - y_M = m(x - x_M) \rightarrow y - 3 = \frac{1}{4}(x - 4) \rightarrow y - 3 = \frac{1}{4}x - 1 \rightarrow y = \frac{1}{4}x + 2$$

۳ برای محاسبه مساحت کافی است طول قاعده ( $BC$ ) و ارتفاع ( $AH$ ) را به دست آوریم.



$$BC = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2}$$

$$BC = \sqrt{(3 - (-1))^2 + (4 - 1)^2} = \sqrt{16 + 9} = 5$$

برای محاسبه  $AH$  باید فاصله نقطه  $A$  از خط گذرنده از  $B$  و  $C$  را تعیین کنیم.

$$B(3, 4), C(-1, 1) : m_{BC} = \frac{4 - 1}{3 - (-1)} = \frac{3}{4}$$

$$y - y_0 = m_{BC}(x - x_0) \xrightarrow{(-1, 1)} y - 1 = \frac{3}{4}(x + 1)$$

$$y = \frac{3}{4}x + \frac{7}{4} \rightarrow \frac{3}{4}x - y + \frac{7}{4} = 0 \rightarrow \begin{cases} a = \frac{3}{4} \\ b = -1 \\ c = \frac{7}{4} \end{cases}$$

$$(0, 1) : AH = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|\frac{3}{4} \times 0 + (-1) \times 1 + \frac{7}{4}|}{\sqrt{\frac{9}{16} + 1}} = \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}$$

$$S = \frac{1}{2}AH \times BC = \frac{1}{2} \times \frac{3}{5} \times 5 = \frac{3}{2}$$

$$n = 1 \rightarrow 2n = 2 \rightarrow 2n + 2 = 4 \rightarrow 2n + 2 = 4 \rightarrow (n + 1)(2) = 0 + 4 \rightarrow \text{نقطه‌ی } A(0, 2) \text{ روی خط } y = x + 4 \text{ قرار دارد.}$$

$$n = 1 \rightarrow 2y = x + 4 \rightarrow y = \frac{1}{2}x + 2 \rightarrow m_1 = \frac{1}{2}$$

$$y = (2m - 7)x + 3 \rightarrow m_2 = 2m - 7$$

$$\text{دو خط عمود} \rightarrow m_1 \cdot m_2 = -1 \rightarrow \frac{1}{2}(2m - 7) = -1 \rightarrow 2m - 7 = -2 \rightarrow 2m = 5 \rightarrow m = \frac{5}{2}$$

طول و ارتفاع وارد بر ضلع  $AB$  = فاصله‌ی نقطه‌ی  $C$  از خط  $AB$  ۵

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{3 - 0}{0 - 4} \rightarrow m_{AB} = -\frac{3}{4}$$

$$y - y_A = m_{AB}(x - x_A) \rightarrow y - 0 = -\frac{3}{4}(x - 4) \rightarrow y = -\frac{3}{4}x + 3$$

$$\rightarrow 4y = -3x + 12 \rightarrow 3x + 4y - 12 = 0 \quad \text{معادله‌ی خط } AB$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|3(1) + 4(1) - 12|}{\sqrt{3^2 + 4^2}} = \frac{5}{5} \rightarrow d = 1 \rightarrow CH = 1$$

$$2x - y + 4 = 0, \quad A(4, 7)$$

$$a = 2, \quad b = -1, \quad c = 4, \quad x_0 = 4, \quad y_0 = 7$$

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|2(4) - 1(7) + 4|}{\sqrt{2^2 + (-1)^2}} = \frac{5}{\sqrt{5}} \rightarrow d = \sqrt{5}$$

$$3x + 4y + 18 = 0 \rightarrow a = 3, \quad b = 4, \quad c = 18$$

$$6x + 8y - 4 = 0 \xrightarrow{\div 2} 3x + 4y - 2 = 0 \rightarrow a = 3, \quad b = 4, \quad c' = -2$$

$$\rightarrow d = \frac{|c - c'|}{\sqrt{a^2 + b^2}} = \frac{|18 - (-2)|}{\sqrt{3^2 + 4^2}} = \frac{20}{5} \rightarrow d = 4$$

$$2x + y - 3 = 0 \xrightarrow{x=3} 2(3) + y - 3 = 0 \rightarrow y = -3 \rightarrow A(3, -3)$$

نقطه‌ی  $A(3, -3)$  روی خط  $(2m - 1)x - y = 0$  قرار دارد ←

$$(2m - 1)(3) - (-3) = 0 \rightarrow 6m - 3 + 3 = 0 \rightarrow 6m = 0 \rightarrow m = 0$$

$$\frac{x^2}{3} - 2 = u \rightarrow u^2 - 7u + 6 = 0 \rightarrow (u - 6)(u - 1) = 0 \begin{cases} u = 6 \\ u = 1 \end{cases}$$

$$u = 6 \rightarrow \frac{x^2}{3} - 2 = 6 \rightarrow \frac{x^2}{3} = 8 \rightarrow x^2 = 24 \rightarrow x = \pm\sqrt{24} \rightarrow \pm 2\sqrt{6}$$

$$u = 1 \rightarrow \frac{x^2}{3} - 2 = 1 \rightarrow \frac{x^2}{3} = 3 \rightarrow x^2 = 9 \rightarrow x = \pm 3$$

$$\alpha\beta = \frac{c}{a} = \frac{4}{1} \rightarrow \alpha\beta = 4$$

$$\alpha\beta^2 + 4 = 0 \rightarrow \alpha\beta(\beta) + 4 = 0 \rightarrow 4\beta + 4 = 0 \rightarrow \beta = -1$$

$$\rightarrow (-1)^2 - 3m(-1) + 4 = 0 \rightarrow 1 + 3m + 4 = 0 \rightarrow 3m = -5 \rightarrow m = -\frac{5}{3}$$

$$X = \frac{1}{x} \rightarrow x = \frac{1}{X} \rightarrow 3\left(\frac{1}{x}\right)^2 - 12\left(\frac{1}{x}\right) = 7 \rightarrow \frac{3}{x^2} - \frac{12}{x} - 7 = 0$$

$$\rightarrow 3 - 12x - 7x^2 = 0 \rightarrow 7x^2 + 12x - 3 = 0$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} \rightarrow S = \alpha + \beta = 3, \quad \alpha\beta = \frac{c}{a} = \frac{-5}{1} \rightarrow P = \alpha\beta = -5$$

$$\frac{1}{\alpha+1} + \frac{1}{\beta+1} = \frac{\beta+1+\alpha+1}{(\alpha+1)(\beta+1)} = \frac{\alpha+\beta+2}{\alpha\beta+\alpha+\beta+1} = \frac{S+2}{P+S+1} = \frac{3+2}{-5+3+1} = \frac{5}{-1} = -5$$

$$x_1 + x_2 = -\frac{b}{a} \rightarrow \begin{cases} x_1 + x_2 = 2 \\ x_1 - x_2 = 2\sqrt{3} \end{cases} +$$

$$2x_1 = 2 + 2\sqrt{3} \rightarrow \boxed{x_1 = 1 + \sqrt{3}}, \quad 1 + \sqrt{3} + x_2 = 2 \rightarrow \boxed{x_2 = 1 - \sqrt{3}}$$

$$\rightarrow x_1 \cdot x_2 = \frac{c}{a} \rightarrow (1 + \sqrt{3})(1 - \sqrt{3}) = \frac{m-1}{1} \rightarrow 1 - 3 = m - 1$$

$$\rightarrow -2 = m - 1 \rightarrow \boxed{m = -1}$$

$a = 1 > 0 \rightarrow$  تابع نقطه‌ی min دارد

$$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = \frac{4}{2} \rightarrow \boxed{x = 2}$$

$$h(2) = 2^2 - 4(2) + 9 = 4 - 8 + 9 = 5 \rightarrow \boxed{y_{\min} = 5}$$

$$\begin{cases} P(0) = 1 \\ \frac{-b}{2a} = 2 \\ P(2) = -1 \end{cases} \rightarrow \begin{cases} \boxed{c = 1} \\ -b = 4a \\ 4a + 2b + c = -1 \end{cases} \rightarrow \begin{cases} -4a - b = 0 \\ 4a + 2b = -2 \end{cases} +$$

$$\boxed{b = -2}, \quad \boxed{a = \frac{1}{2}}$$

$$\rightarrow \boxed{P(x) = \frac{1}{2}x^2 - 2x + 1}$$

$$\begin{cases} P(0) = 3 \\ \frac{-b}{2a} = -4 \\ P(-4) = -2 \end{cases} \rightarrow \begin{cases} \boxed{c = 3} \\ b = 8a \\ 16a - 4b + c = -2 \end{cases} \rightarrow \begin{cases} b - 8a = 0 \\ 16a - 4b = -5 \end{cases}$$

$$\rightarrow \begin{cases} 4b - 32a = 0 \\ 16a - 4b = -5 \end{cases} +$$

$$-16a = -5 \rightarrow \boxed{a = \frac{5}{16}}, \quad \boxed{b = \frac{5}{2}}$$

$$\rightarrow \boxed{P(x) = \frac{5}{16}x^2 + \frac{5}{2}x + 3}$$

$a > 0, \quad \Delta > 0 \rightarrow$  دو ریشه‌ی حقیقی متمایز

$$\frac{c}{a} < 0 \rightarrow c < 0$$

$$-\frac{b}{a} > 0 \rightarrow b < 0$$

**الف**

$$\rightarrow \sqrt{x+1} - 1 = \sqrt{2x-5} \rightarrow (\sqrt{x+1} - 1)^2 = (\sqrt{2x-5})^2$$

$$\rightarrow x + 1 - 2\sqrt{x+1} + 1 = 2x - 5 \rightarrow x + 2 - 2\sqrt{x+1} = 2x - 5$$

$$\rightarrow -x + 7 = 2\sqrt{x+1} \rightarrow (-x+7)^2 = (2\sqrt{x+1})^2$$

$$\rightarrow x^2 - 14x + 49 = 4(x+1) \rightarrow x^2 - 18x + 45 = 0$$

$$\rightarrow (x-3)(x-15) = 0 \begin{cases} x-3=0 \rightarrow \boxed{x=3} \\ x-15=0 \rightarrow x=15 \text{ غير قابل قبول} \end{cases}$$

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**الف**

$$\rightarrow \frac{2}{k} - \frac{3k}{k+2} = \frac{k}{k(k+2)} \rightarrow \text{مخرجها} = k(k+2) \begin{cases} k \neq 0 \\ k \neq -2 \end{cases}$$

$$\rightarrow k(k+2) \times \left[ \frac{2}{k} - \frac{3k}{k+2} = \frac{k}{k(k+2)} \right]$$

$$\rightarrow 2(k+2) - 3k^2 = k \rightarrow 2k + 4 - 3k^2 = k \rightarrow 3k^2 - k - 4 = 0$$

$$\rightarrow \Delta = (-1)^2 - 4(3)(-4) = 1 + 48 = 49$$

$$\rightarrow k = \frac{1 \pm \sqrt{49}}{2(3)} = \frac{1 \pm 7}{6} \begin{cases} k = \frac{4}{3} \\ k = -1 \end{cases}$$

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**الف**

$$\sqrt{15 + \sqrt{2x+80}} = 5$$

$$\xrightarrow{\text{طرفین به توان ۲}} 15 + \sqrt{2x+80} = 25 \rightarrow \sqrt{2x+80} = 10 \xrightarrow{\text{طرفین به توان ۲}} 2x+80 = 100 \rightarrow x = 10 \text{ قی}$$

**ب**

$$\sqrt{x+3} - \sqrt{3x+6} = 1 \rightarrow \sqrt{x+3} = 1 + \sqrt{3x+6} \xrightarrow{\text{طرفین به توان ۲}} x+3 = 1 + 2\sqrt{3x+6} + 3x+6 \rightarrow -2x-4 = 2\sqrt{3x+6}$$

$$\rightarrow -x-2 = \sqrt{3x+6} \xrightarrow{\text{طرفین به توان ۲}} x^2 + 4x + 4 = 3x+6 \rightarrow x^2 + x - 2 = 0 \rightarrow \begin{cases} x = 1 \text{ قی} \\ x = -2 \text{ غی} \end{cases}$$